INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & MANAGEMENT

On intuitionistic fuzzy regular generalized Semipreclosed sets

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ABSTRACT
In this paper, we introduce the notion of intuitionistic fuzzy regular generalized semipreclosed sets. Furthermore, we investigate some of the properties and characterizations of the intuitionistic fuzzy regular generalized semipreclosed sets.
Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy semipreclosed sets, Intuitionistic fuzzy regular generalized semipreclosed sets.

1 Introduction

Ever since the establishment of fuzzy sets by Zadeh [13] in 1965, fuzzy has invaded almost all branches of Mathematics. Later the introduction of fuzzy topology by Chang [3] in 1967 was an annexation towards the hike of fuzzy sets and fuzzy topology. The perception of intuitionistic fuzzy sets by Atanassov [2] was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using the notion of intuitionistic fuzzy sets, Coker [4] has constructed the basic concepts of intuitionistic fuzzy topological spaces. Subsequently in 1986 Andrijevic [1] proposed the notion of semipreclosed sets in general topologywhich was followed by the introduction of intuitionistic fuzzy generalized semipreclosed sets by R. Santhi and D. Jayanthi [8] in 2010. We now extend our visionary towards intuitionistic fuzzy regular generalized semipreclosed sets and study some of their properties and characterizations.

2 Preliminaries

Definition 2.1 [2]:An *intuitionistic fuzzy set* (IFS in short) A is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$where the function $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2 [2] : Let A and B be two IFSs of the form$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$;

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [4] :An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0\sim , 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The compliment A^c of an IFOS A in IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 [4] : Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* are defined by

$int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$

$cl(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5 [6] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$

(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $cl(int(A)) \subseteq A$

(iii) *intuitionistic fuzzy α closed set* (IFαCS in short) if $cl(int(cl(A))) \subseteq A$

(iv) *intuitionistic fuzzy β closed set* (IFβCS in short) if $int(cl(int(A))) \subseteq A$

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFSCSs, IFPCSs, IFαCSs and IFβCSs (respectively IFSOSs, IFPOSs, IFαOSs and IFβOSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X), IFαC(X), IFβC(X) (respectively IFSO(X), IFPO(X), IFαO(X), IFβO(X)).

Definition 2.6 [13] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) *intuitionistic fuzzy semi-pre closed set* (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$,

(ii) *intuitionistic fuzzy semi-pre open set* (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not hold in general.

Definition 2.7 [11] : An IFS A is an

(i) *intuitionistic fuzzy regular closed set* (IFRCS in short) if $A = cl(int(A))$

(ii) *intuitionistic fuzzy regular open set* (IFROS in short) if $A = int(cl(A))$

(iii) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(iv) *intuitionistic fuzzy regular generalized closed set* (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFROS.

Definition 2.8 [5] : An *intuitionistic fuzzy point* (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \le \mu_A$ and $\beta \ge \nu_A$.

Definition 2.9 [10] : Two IFSs are said to be *q-coincident* ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.10 [10] : Two IFSs are said to be *not q-coincident* ($A\, {}_q^c\, B$ in short) if and only if $A \subseteq B^c$.

3 Intuitionistic fuzzy regular generalized semipreclosed sets

In this section we introduce the notion of intuitionistic fuzzy regular generalized semipreclosed sets and study some of their properties including the relation between intuitionistic fuzzy regular generalized semipreclosed sets and few of the other already existing intuitionistic fuzzy sets.

Definition 3.1 An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy regular generalized semipreclosed set* (IFRGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in (X,τ).

The family of all IFRGSPCSs of an IFTS (X,τ) is denoted by IFRGSPC(X).

Example 3.2 Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Now spcl(A) = A. We have A⊆ G. Hence spcl(A) ⊆ G, where G is an IFROS in X. This implies that A is an IFRGSPCS in X.

Theorem 3.3 Every IFCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFCS then cl(A) = A. Let A⊆ U and U is an IFROS. Then spcl(A) ⊆ cl(A) = A ⊆ U, by hypothesis, A is an IFRGSPCS.

Theorem 3.4 Every IFGCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A⊆ U and U be an IFROS. Since every IFROS is an IFOS, U is an IFOS in X. Then by hypothesis cl(A) ⊆ U. As spcl(A) ⊆ cl(A) we have spcl(A) ⊆ U. Hence A is an IFRGSPCS.

Theorem 3.5 Every IFSPCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFSPCS and A ⊆ U, where U is an IFROS. Then since spcl(A) = A and A ⊆ U, we have spcl(A) ⊆ U. Hence A is an IFRGSPCS.

Theorem 3.6 Every IFβCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFβCS and A ⊆ U, U is an IFROS. Then since βcl(A) = A and A ⊆ U, we have βcl(A) ⊆ U. Hence A is an IFRGSPCS.

Theorem 3.7 Every IFSCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFSCS. Since every IFSCS is an IFSPCS[12], by Theorem 3.5, A is an IFRGSPCS.

Theorem 3.8 Every IFPCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFPCS. Since every IFPCS is an IFSPCS[12], by Theorem 3.5, A is an IFRGSPCS.

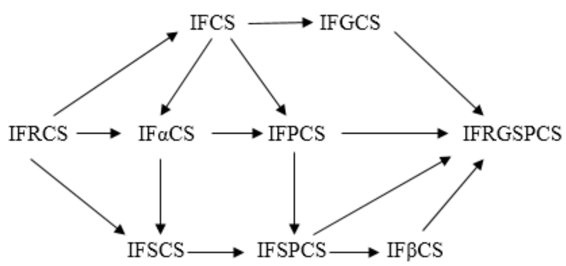
Theorem 3.9 Every IFαCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFαCS. Since every IFαCS is an IFSPCS [12] , by Theorem 3.5, A is an IFRGSPCS.

Theorem 3.10 Every IFRCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFRCS. Since every IFRCS is an IFCS [10], by Theorem 3.3, A is an IFRGSPCS.

Remark 3.11 Every IFCS, IFPCS, IFSCS, IFRCS, IFGCS, IFSPCS, IFαCS, IFβCS are IFRGSPCS but their converses need not be true in general.



In the above diagram the reverse implications are not true in general. This can be easily seen from the following examples.

Example 3.12 Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. As spcl(A) = A, we have A ⊆ G implies spcl(A) ⊆ G, where G is an IFROS in X. This implies that A is an IFRGSPCS in X. Now since cl(A) = $G^c \neq$ A, A is not an IFCS in X. Also A ⊆ G but cl(A) =

$G^c \not\subseteq G$. Therefore A is not an IFGCS in X. Now cl(int(A)) = cl(0~) = 0~ ≠ A. Therefore A is not an IFRCS in X. Hence A is an IFRGSPCS but not IFCS, IFGCS, IFRCS.

Example 3.13 Let X = {a, b} and G = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.4$. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. As spcl(A) = 1~, we have A ⊆ 1~ implies spcl(A) ⊆ 1~, where 1~ is an IFROS. This implies that A is an IFRGSPCS in X. Now since cl(int(A) = cl(G) = 1~ ⊈ A we get A is not an IFPCS in X. Further int(cl(int(A))) = int(1~) = 1~ ⊈ A. Hence A is not an IFβCS in X. Also int(cl(A)) = int(1~) = 1~ ⊈ A. Thus A is not an IFSCS in X. Now since cl(int(cl(A))) = cl(1~) = 1~ ⊈ A, A is not an IFαCS in X. Further there exists no IFPCS B such that int(B) ⊆ A ⊆ B. Therefore A is not an IFSPCS in X. Hence A is an IFRGSPCS but not IFPCS, IFβCS, IFSCS, IFαCS, IFSPCS.

Theorem 3.14 Let (X,τ) be an IFTS. Then for every A ∈ IFRGSPC(X) and for every B ∈ IFS(X), A ⊆ B ⊆ spcl(A) ⇒ B ∈ IFRGSPC(X).

Proof: Let B ⊆ U and U be an IFROS. Then since, A⊆ B, A ⊆ U. By hypothesis,B ⊆ spcl(A). Therefore spcl(B) ⊆ spcl(spcl(A)) = spcl(A) ⊆ U, since A is an IFRGSPCS. Hence B ∈ IFRGSPC(X).

Theorem 3.15 An IFS A of an IFTS (X,τ) is an IFRGSPCS if and only if A $_q{}^c$ F ⇒ spcl(A) $_q{}^c$ F for every IFRCS F of X.

Proof: Necessity: Let F be an IFRCS and A $_q{}^c$ F, then by Definition 2.10, A ⊆ F^c, where F^c is an IFROS. Then spcl(A) ⊆ F^c, by hypothesis. Hence again by Definition 2.10, spcl(A) $_q{}^c$ F.

Sufficiency: Let U be an IFROS such that A ⊆ U. Then U^c is an IFRCS and A ⊆ $(U^c)^c$. By hypothesis , A $_q{}^c$ U^c ⇒ spcl(A) $_q{}^c$ U^c. Hence by Definition 2.10, spcl(A) ⊆ $(U^c)^c$ = U. Therefore spcl(A) ⊆ U. Hence A is an IFRGSPCS.

Theorem 3.16 Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFRGSPCS if and only if IFSPO(X) = IFSPC(X).

Proof: Necessity: Suppose that every IFS in (X,τ) is an IFRGSPCS. Let U ∈ IFRO(X), then U ∈ IFSPO(X) and by hypothesis, spcl(U) ⊆ U ⊆ spcl(U). This implies spcl(U) = U. Therefore U ∈ IFSPC(X).Hence IFSPO(X) ⊆ IFSPC(X). Let A ∈ IFSPC(X), then A^c ∈ IFSPO(X) ⊆ IFSPC(X). That is, A^c ∈ IFSPC(X). Therefore A∈ IFSPO(X). Hence IFSPC(X) ⊆ IFSPO(X). Thus IFSPO(X) = IFSPC(X).

Sufficiency: Suppose that IFSPO(X) = IFSPC(X). Let A⊆ U and U be an IFROS. Then U ∈ IFSPO(X) and spcl(A) ⊆ spcl(U) = U, since U ∈ IFSPC(X), by hypothesis. Therefore A is an IFRGSPCS in X.

Theorem 3.17 If A is an IFROS and an IFRGSPCS in (X,τ) then A is an IFSPCS in (X,τ).

Proof: Since A ⊆ A and A is an IFROS, by hypothesis, spcl(A) ⊆ A. But A ⊆ spcl(A). Therefore spcl(A) = A. Hence A is an IFSPCS.

Theorem 3.18 Let A be an IFRGSPCS in (X,τ) and p(α, β) be an IFP in X such that int(p(α, β)) $_q$ spcl(A), then cl(int(p(α, β))) $_q$ A.

Proof: Let A be an IFRGSPCS and let int(p(α, β)) $_q$ spcl(A). If cl(int(p(α, β))) $_q{}^c$ A then by Definition 2.10, A ⊆ $[cl(int(p(α, β)))]^c$ where $[cl(int(p(α, β)))]^c$ is an IFROS. Then by hypothesis, spcl(A) ⊆ $[cl(int(p(α, β)))]^c$ = int(cl($p(α, β)^c$)) ⊆ cl($p(α, β)^c$) =$(int(p(α, β)))^c$. This implies that spcl(A) ⊆ $(int(p(α, β)))^c$. Therefore by Definition 2.10, int(p(α, β)) $_q{}^c$ spcl(A), which is a contradiction to the hypothesis. Hence cl(int(p(α, β))) $_q$ A.

Theorem 3.19 Let F ⊆ A ⊆ X where A is an IFROS and an IFRGSPCS in X. Then F is an IFRGSPCS in A if and only if F is an IFRGSPCS in X.

Proof: Necessity: Let U be an IFROS in X and F ⊆ U. Also let F be an IFRGSPCS in A. Then clearly F ⊆A∩ U and A ∩ U is an IFROS in A. Hence the semipre closure of F in A, $spcl_A(F)$ ⊆ A ∩ U. By Theorem 3.17, A is an IFSPCS. Therefore spcl(A) = A and the semipre closure of F in X, spcl(F) ⊆ spcl(F) ∩ spcl(A) = spcl(F) ∩ A = $spcl_A(F)$ ⊆ A ∩ U ⊆ U. That is, spcl(F) ⊆ U whenever F ⊆ U. Hence F is an IFRGSPCS in A.

Sufficiency: Let V be an IFROS in A such that F ⊆ V. Since A is an IFROS in X ,Vis an IFROS in X. Therefore spcl(F) ⊆ V, since F is an IFRGSPCS in X. Thus $spcl_A(F)$ = spcl(F) ∩ A ⊆ V ∩ A ⊆ V. Hence F is an IFRGSPCS in A.

Theorem 3.20 Let (X, τ) be an IFTS, then for every A ∈ IFSPC(X) and for every IFS B in X, int(A) ⊆ B ⊆ A ⇒ B ∈ IFRGSPC(X).

Proof: Let A be an IFSPCS in X. Then by Definition 2.6, there exits an IFPCS, say C such that int(C) ⊆ A ⊆ C. By hypothesis, B ⊆ A.Therefore B ⊆ C. Since int(C) ⊆ A, int(C) ⊆ int(A) and int(C) ⊆ B. Thus int(C) ⊆ B ⊆ C and by Definition 2.6, B ∈ IFSPC(X). Hence by Theorem 3.5, B ∈ IFRGSPC(X).

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